Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 6: Linear Equations in One Variable

Definition 1. **Equation:** *An equation is an equality between two mathematical expressions. For example,* $7 - 5 = 2$ *is an equation.*

Definition 2. *An* **equation in one variable** *is a statement that two expressions, with at least one containing the variable, are equal. For example,* $2x - 3 = 7$ *is an equation in the variable* x*.*

Definition 3. **Domain:** *The* **domain of the variable** *in an equation is the set of all real numbers for which both sides of the equation are defined.*

Example 1.

1. *The domain of* $2x - 3 = 7$ *is all real numbers.*
2. *The domain of the equation* $\frac{24}{x} - 3 = \frac{18}{x}$ *is all real numbers* *except* 0*.*

Definition 4. **Inconsistent Equation:** *An equation that has no solution is called an inconsistent equation. For example, the equation* $x = x + 5$ *is an inconsistent equation.*

Definition 5. **Equivalent Equations:** *Equations that have the same solution set are called equivalent equations. For example, equations* $x = 4$ *and* $3x = 12$ *are equivalent equations with solution set* $\{4\}$*.*

Example 2. *Solve the following linear equations:*
$\frac{1}{3}x - \frac{2}{3} = \frac{1}{2}(1 - 3x)$ *and* $1 - 5y + 7(2 + y) = 2y - 5(3 - y)$*.*
Solution:

$$\begin{aligned}
\frac{1}{3}x - \frac{2}{3} &= \frac{1}{2}(1 - 3x) \\
6(\frac{1}{3}x - \frac{2}{3}) &= 6 \cdot \frac{1}{2}(1 - 3x) \\
6 \cdot \frac{1}{3}x - 6 \cdot \frac{2}{3} &= 3(1 - 3x) \\
2x - 4 &= 3 - 9x \\
2x - 4 + 9x &= 3 - 9x + 9x \\
11x - 4 &= 3 \\
11x - 4 + 4 &= 3 + 4 \\
11x &= 7 \\
x &= \frac{7}{11}
\end{aligned}$$

$$\begin{aligned}
1 - 5y + 7(2 + y) &= 2y - 5(3 - y) \\
1 - 5y + 7 \cdot 2 + 7y &= 2y - 5 \cdot 3 - 5(-y) \\
1 + 14 + 2y &= 2y - 15 + 5y \\
15 + 2y &= -15 + 7y \\
15 + 2y + 15 &= -15 + 7y + 15
\end{aligned}$$

$$\begin{aligned}30 + 2y &= 7y \\ 30 + 2y - 2y &= 7y - 2y \\ 30 &= 5y \\ y &= \frac{30}{5} = 6\end{aligned}$$

Example 3. *Solve the formula* $A = \frac{(a+b)h}{2}$ *for* h.
Solution:

$$\begin{aligned}A &= \frac{(a+b)h}{2} \\ 2 \cdot A &= (a+b)h \\ \frac{2A}{(a+b)} &= h\end{aligned}$$

So $h = \frac{2A}{(a+b)}$

Example 4. *Solve the formula* $M = H + \frac{8}{T}$ *for* T.
Solution:

$$\begin{aligned}M &= H + \frac{8}{T} \\ M - H &= H + \frac{8}{T} - H \\ M - H &= \frac{8}{T} \\ \frac{1}{M-H} &= \frac{T}{8} \\ 8 \cdot \frac{1}{M-H} &= T \\ \frac{8}{M-H} &= T\end{aligned}$$

So $T = \frac{8}{M-H}$.